Modeling sediment transport in shallow waters

Tomás Morales de Luna

Sediments are solid fragmented materials, such as silt, sand, gravel, chemical precipitates, and fossil fragments, that are transported and deposited by water, ice, or wind or that accumulates through chemical precipitation or secretion by organisms, and that forms layers on the Earth's surface. The study of sediment transport processes includes movement of rocks in a mountain as material diffusion in water, among other processes. The focus here is to study sediment transport by the action of some fluid such as in an ocean or a river. Accurate prediction of sediment transport rates is an important element in morphological studies of river, coastal, and marine environments.

Sediment particles are transported by flow in one or a combination of ways: rolling or sliding on the bed, surface creep; jumping into the flow and then resting on the bed, saltation; and supported by the surrounding fluid during significant part of its motion, suspension. Nevertheless, sediment transport is usually classified in two main modes: bed load and suspended load. The bed load is the part of the total load which is travelling immediately above the bed and is supported by intergranular collisions rather than fluid turbulence. The suspended load, on the other hand, is the part of the load which is primarily supported by the fluid turbulence

Concerning bedload transport, one possible approach to model this type of transport is to use a coupled system consisting on a hydrodynamical component, which describes the behavior of the fluid and modeled using shallow water equations, and a morphodynamical component, which describes the transport of the sediment and modeled by a transport equation. This leads to the Exner system [**?**]:

$$\begin{cases} \partial_t h + \partial_x(hu) = 0, \\ \partial_t(hu) + \partial_x(hu^2 + g\dfrac{h^2}{2}) + gh\partial_x(z_b + z_f) = 0, \\ \partial_t z_b + \xi\partial_x q_b(h, hu, z_b) = 0. \end{cases}$$

The thickness of the fluid is denoted by h, and u represents the horizontal velocity. The thickness of the sediment layer that is subject to bedload transport is given by z_b. The porosity of the sediment layer is given by γ, where $\xi = (1-\gamma)^{-1}$. Finally, the model depends on the definition of the solid transport flux q_b which is defined empirically. Among the most well-known formulas are the ones given by Grass [3] and Meyer-Peter & Müller [4].

This model may be completed and generalized including suspended sediment transport and other more complex effects (See [5, 1, 2]). More recent techniques use a multilayer approach to better describe the process (See [6]).

All these models may be written under the formalism of hyperbolic systems of conservation laws with source terms and non-conservative products. The numerical simulations of such systems may be done under the general framework of path-conservative numerical schemes [7].

References

[1] E. D. Fernández-Nieto, C. Lucas, T. Morales de Luna, and S. Cordier. On the influence of the thickness of the sediment moving layer in the definition of the bedload transport formula in exner systems. *Computers & Fluids*, 91:87–106, Mar. 2014.

[2] E. D. Fernández-Nieto, T. Morales de Luna, G. Narbona-Reina, and J. d. D. Zabsonré. Formal deduction of the Saint-Venant-Exner model including arbitrarily sloping sediment beds and associated energy. *ESAIM: Mathematical Modelling and Numerical Analysis*, 51(1):115–145, Jan. 2017.

[3] A. Grass. Sediment transport by waves and currents. *SERC London Cent. Mar. Technol*, Report No. FL29, 1981.

[4] E. Meyer-Peter and R. Müller. Formulas for bed-load transport. In *2nd meeting IAHSR, Stockholm, Sweden*, pages 1–26, 1948.

[5] T. Morales de Luna, M. J. Castro Díaz, C. Parés Madroñal, and E. D. Fernández Nieto. On a shallow water model for the simulation of turbidity currents. *Communications in Computational Physics*, 6(4):848–882, 2009.

[6] T. Morales de Luna, E. D. Fernández Nieto, and M. J. Castro Díaz. Derivation of a multilayer approach to model suspended sediment transport: Application to hyperpycnal and hypopycnal plumes. *Communications in Computational Physics*, 22(5):1439–1485, 2017.

[7] C. Parés. Numerical methods for nonconservative hyperbolic systems: a theoretical framework. *SIAM J. Numer. Anal.*, 44(1):300–321 (electronic), 2006.